Soliton wave solutions of stochastic RKL equation with multiplicative white noise using the generalized (F'/F) -expansion approach

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Abstract:
This article explores soliton wave solutions for the system of the stochastic Radhakrishnan-Kundu-Lakshmanan (R.K.L) equations within magneto-optic waveguides. The system incorporates multiplicative white noise in the Ito sense and power law nonlinearity. The generalized (F'/F) -expansion approach, along with the auxiliary equation $F'^2(\xi) = R_0 + R_2F^2(\xi) + R_4F^4(\xi)$, is utilized to derive exact optical wave solutions. The study presents a variety of soliton wave solutions, including dark (kink) and singular solitons, as well as Jacobi-elliptic functions wave solutions. Finally, a comparison is made between our findings and existing literature.

Keywords: *Radhakrishnan-Kundu-Lakshmanan equation; White noise; Generalized (F'/F) -expansion approach; Magneto-optics waveguides.*

1. Introduction

systems of nonlinear evolution equations are well-known for their importance in modeling various phenomena across the physical sciences, such as nonlinear optical fibers, waveguides, plasma physics, quantum optics, fluid dynamics, and telecommunications. As a result, there has been increasing interest from researchers in finding soliton wave solutions for these nonlinear partial differential equations (PDEs), leading to the exploration of various methods. Recent developments have delved into topics like dispersive solitons, highly-dispersive solitons, pure-cubic solitons, cubic-quartic solitons, dispersion-managed solitons, as well as the effects of white noise and magneto-optic waveguides, among many other related areas [1-13]. This study specifically focuses on optical solitons in dispersive media, which have been analyzed in several models, including the Fokas-Lenells equation [14] and the Schrödinger-Hirota equation [15], among others. This article primarily focuses on dispersive solitons within the framework of the well-established R.K.L equation [15-26]. In this research, we examine the cubic system of magneto-optic waveguides associated with the nonlinear R.K.L equation, incorporating nonlinear power law and multiplicative white noise [26].

$$i\varphi_t + a_1\varphi_{xx} + (b_1|\varphi|^{2n} + c_1|\psi|^{2n})\varphi + i[\beta_1\varphi_{xxx} + \alpha_1(|\varphi|^{2n}\varphi)_x] + \sigma\varphi W_t(t) = Q_1\psi + i[\lambda_1\varphi_x + \mu_1(|\varphi|^{2n})_x\varphi + \theta_1|\varphi|^{2n}\varphi_x], \quad (1)$$

and

$$i\psi_t + a_2\psi_{xx} + (b_2|\psi|^{2n} + c_2|\varphi|^{2n})\psi + i[\beta_2\psi_{xxx} + \alpha_2(|\psi|^{2n}\psi)_x] + \sigma\psi W_t(t) = Q_2\varphi + i[\lambda_2\psi_x + \mu_2(|\psi|^{2n})_x\psi + \theta_2|\psi|^{2n}\psi_x], \quad (2)$$

Here, $\varphi(x,t)$ and $\psi(x,t)$ represent complex-valued functions that characterize the wave profiles, while $a_j,\ b_j,\ c_j,\ \beta_j,\ \alpha_j,\ \sigma,\ Q_j,\ \lambda_j,\ \mu_j,\ \theta_j\ \ (j = 1,2)$ are real constants. $Q_j\ (j = 1,2)$ are the coefficients of magneto-optic waveguides terms. a_j are the coefficients of chromatic dispersion, $b_j,\ c_j$ are the coefficients of self-phase modulation and cross-phase modulation respectively. β_j are the coefficients of third order dispersion. λ_j are the coefficients of the intermodal dispersion, $\alpha_j,\ \mu_j$ and θ_j are the coefficients of nonlinear dispersion terms. The solitary wave solutions for Equations (1) and (2) are obtained through the generalized (F'/F) -expansion approach.

This article is structured as follows: Section 2 offers a mathematical analysis of Equations (1) and (2). In Section 3, we will derive the optical solitons for these equations using the generalized (F'/F) -expansion approach. Finally, Section 4 presents the conclusions derived from the study.

2. Converting to ordinary differential equations

For solving Equations (1) and (2), we suppose that the wave profiles take the forms:

$$\varphi(x,t) = U_1(\xi)e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \quad (3)$$
$$\psi(x,t) = U_2(\xi)e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \quad (4)$$

$$\xi = (x-\rho t), \vartheta(x,t) = -\kappa x + \omega t, \quad (5)$$

where ρ, κ and ω are real constants, and the functions $\vartheta(x,t)$, $U_j(\xi)$ $(j=1,2)$ are real functions. the constant κ represents the frequency, while the constant ρ represents the velocity and the constant ω represents the wave number. The function $\vartheta(x,t)$ is the phase component. Finally, the functions $U_j(\xi)$ $(j=1,2)$ are the amplitude components. By evaluating (3), (4) into Equations (1), (2), the real parts are derived as follows:

$$(3\kappa\beta_1 + a_1)U_1'' + [(\alpha_1 - \theta_1)\kappa + b_1]U_1^{2n+1} + c_1 U_1 U_2^{2n}$$
$$-(\kappa^3\beta_1 + \kappa^2 a_1 + \kappa\lambda_1 - \sigma^2 + \omega)U_1 - Q_1 U_2 = 0, \quad (6)$$

and

$$(3\kappa\beta_2 + a_2)U_2'' + [\kappa(\alpha_2 - \theta_2) + b_2]U_2^{2n+1} + c_2 U_2 U_1^{2n}$$
$$-(\kappa^3\beta_2 + \kappa^2 a_2 + \kappa\lambda_2 - \sigma^2 + \omega)U_2 - Q_2 U_1 = 0. \quad (7)$$

Also, we can derive the imaginary parts as follows:

$$\beta_1 U_1''' - [2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1]U_1^{2n}U_1' - (3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1)U_1' = 0, \quad (8)$$

and

$$\beta_2 U_2''' - [2n(\mu_2 - \alpha_2) + \theta_2 - \alpha_2]U_2^{2n}U_2' - (3\beta_2\kappa^2 + 2a_2\kappa + \rho + \lambda_2)U_2' = 0. \quad (9)$$

For simplicity, we can let

$$U_2(\xi) = \Omega_1 U_1(\xi), \quad (10)$$

where Ω_1 is a constant, such that $\Omega_1 \neq 0$ and $\Omega_1 \neq 1$. Equations (6), (7), (8) and (9) can be reduced as:

$$(3\kappa\beta_1 + a_1)U_1'' + [(\alpha_1 - \theta_1)\kappa + \Omega_1^{2n}c_1 + b_1]U_1^{2n+1}$$
$$-(\kappa^3\beta_1 + \kappa^2 a_1 + \kappa\lambda_1 - \sigma^2 + \omega + \Omega_1 Q_1)U_1 = 0, \quad (11)$$

$$(3\kappa\beta_2 + a_2)U_1'' + [\Omega_1^{2n}(\kappa(\alpha_2 - \theta_2) + b_2) + c_2]U_1^{2n+1}$$
$$-\left(\kappa^3\beta_2 + \kappa^2 a_2 + \kappa\lambda_2 - \sigma^2 + \omega + \frac{Q_2}{\Omega_1}\right)U_1 = 0, \quad (12)$$

$$\beta_1 U_1''' - [2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1]U_1^{2n}U_1' - (3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1)U_1' = 0, \quad (13)$$

$$\beta_2 U_1''' - \Omega_1^{2n}[2n(\mu_2 - \alpha_2) + \theta_2 - \alpha_2]U_1^{2n}U_1' - (3\beta_2\kappa^2 + 2a_2\kappa + \rho + \lambda_2)U_1' = 0. \quad (14)$$

On applying the principle of linear independence on Equations (11) and (12), we get:

$$\kappa = -\frac{a_1}{3\beta_1} \text{ or } \kappa = -\frac{a_2}{3\beta_2}, \quad (15)$$

and

$$\left.\begin{array}{l}\omega = \sigma^2 - (\kappa^3\beta_1 + \kappa^2 a_1 + \kappa\lambda_1 + \Omega_1 Q_1) \\ \text{or } \omega = \sigma^2 - \left(\kappa^3\beta_2 + \kappa^2 a_2 + \kappa\lambda_2 + \frac{Q_2}{\Omega_1}\right)\end{array}\right\}, \quad (16)$$

as well as the parametric restrictions:

$$\left.\begin{array}{r}(\alpha_1 - \theta_1)\kappa + \Omega_1^{2n}c_1 + b_1 = 0 \\ \Omega_1^{2n}[\kappa(\alpha_2 - \theta_2) + b_2] + c_2 = 0\end{array}\right\}, \quad (17)$$

provided $a_j \neq 0$ and $\beta_j \neq 0$ $(j=1,2)$.

Equations (13) and (14) are the same form under the following conditions:

$$\frac{\beta_1}{\beta_2} = \frac{2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1}{\Omega_1^{2n}[2n(\mu_2 - \alpha_2) + \theta_2 - \alpha_2]} = \frac{3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1}{3\beta_2\kappa^2 + 2a_2\kappa + \rho + \lambda_2}. \quad (18)$$

From Equations given by (18), we can determine the soliton velocity as:

$$\rho = \frac{\beta_2(2\kappa a_1 + \lambda_1) - \beta_1(2a_2\kappa + \lambda_2)}{\beta_1 - \beta_2}, \quad (19)$$

provided $\beta_1 \neq \beta_2$.

Now, let us solve Equation (13) under the conditions (18). To this aim, we integrate Equation (13) and put the integration constant equal to zero

$$\beta_1 U_1'' - \left[\frac{2n(\mu_1 - \alpha_1) + \theta_1 - \alpha_1}{2n+1}\right]U_1^{2n+1} - (3\beta_1\kappa^2 + 2a_1\kappa + \rho + \lambda_1)U_1 = 0, \quad (20)$$

By considering U_1'' and U_1^{2n+1} in Equation (20), we determine $N = \frac{1}{n}$. Subsequently, the transformation:

$$U_1(\xi) = [V(\xi)]^{\frac{1}{n}}, \quad (21)$$

where $V(\xi) > 0$ is a function of ξ and $n > 1$. Putting (21) into Equation (20) changes it to the new ODE:

$$\beta_1(2n+1)[nV''V - (n-1)V'^2] - n^2(2n+1)(3\kappa^2\beta_1 + 2\kappa a_1 + \rho + \lambda_1)V^2$$
$$+n^2[2n(\alpha_1 - \mu_1) + \alpha_1 - \theta_1]V^4 = 0. \quad (22)$$

Next, we will apply the generalized (F'/F)-expansion approach to determine the solitons waves and other exact solutions of Equations (1), (2).

3. The generalized (F'/F)-expansion approach

By balancing VV'' and V^4 in equation (22), we obtain the balance number $N = 1$. The (F'/F) expansion approach [27-29] assumes the exact wave solution of equation (22) can be written as:

$$V(\xi)=A_0+A_1\left[\frac{F'(\xi)}{F(\xi)}\right], \qquad (23)$$

and $F(\xi)$ is a function of ξ satisfying the Jacobi elliptic equation:

$$F'^2(\xi)=R_0+R_2F^2(\xi)+R_4F^4(\xi), \qquad (24)$$

where $A_0,\ A_1,\ R_0,\ R_2$ and R_4 are constants, such that $A_1\neq 0$. Equation (24) has many exact solutions of Jacobi-elliptic functions and Weierstrass-elliptic functions [27-31] as the following tables:

Table A: (The Jacobi-elliptic function solutions) :

Case	R_4	R_2	R_0	$F(\xi)$
1	l^2	$-(l^2+1)$	1	sn (ξ,l)
2	l^2	$-(1+l^2)$	1	cd $(\xi,l)=\frac{cn(\xi,l)}{dn(\xi,l)}$
3	$-l^2$	$2l^2-1$	$1-l^2$	$cn(\xi,l)$
4	$1-l^2$	$2-l^2$	1	$sc(\xi,l)=\frac{sn(\xi,l)}{cn(\xi,l)}$
5	$\frac{1}{4}$	$\frac{1-2l^2}{2}$	$\frac{1}{4}$	$ns(\xi,l)\pm cs(\xi,l)$

By differentiating equation (23) and successively applying equation (24), we can get the following derivatives:

$$\begin{cases} V'^2(\xi)=A_1^2R_2^2-2A_1^2R_2\left(\frac{F'(\xi)}{F(\xi)}\right)^2+A_1^2\left(\frac{F'(\xi)}{F(\xi)}\right)^4-4R_4A_1^2R_0 \\ V''(\xi)=2A_1\left(\frac{F'(\xi)}{F(\xi)}\right)\left[\left(\frac{F'(\xi)}{F(\xi)}\right)^2-R_2\right]. \end{cases} \qquad (25)$$

By substituting (23) and (25) into equation (22), then collecting all the coefficients of $\left(\frac{F'(\xi)}{F(\xi)}\right)^i$, $(i=0,\ 1,\ 2,\ 3,\ 4)$, setting them equal to zero, we get the algebraic equations:

$\left(\frac{F'(\xi)}{F(\xi)}\right)^4$: $2\beta(2\vartheta_1^2{}_1n-\vartheta_1^2(n-1))\left(n+\frac{1}{2}\right)+2n^2\left((\alpha_1-\mu_1)n+\frac{\alpha_1}{2}-\frac{\theta_1}{2}\right)\vartheta_1^4=0,$

$\left(\frac{F'(\xi)}{F(\xi)}\right)^3$: $4\beta_1\vartheta_1\vartheta_0 n\left(n+\frac{1}{2}\right)+8n^2\left((\alpha_1-\mu_1)n+\frac{\alpha_1}{2}-\frac{\theta_1}{2}\right)\vartheta_1^3\vartheta_0=0,$

$\left(\frac{F'(\xi)}{F(\xi)}\right)^2$: $2\beta_1\left(-2\vartheta_1^2R_2n+2\vartheta_1^2R_2(n-1)\right)\left(n+\frac{1}{2}\right)+12n^2\left((\alpha_1-\mu_1)n+\frac{\alpha_1}{2}-\frac{\theta_1}{2}\right)\vartheta_0^2\vartheta_1^2$
$-2n^2\left(n+\frac{1}{2}\right)(3\kappa^2\beta_1+2\kappa a_1+\rho+\lambda_1)\vartheta_1^2=0,$

$\left(\frac{F'(\xi)}{F(\xi)}\right)$: $-4\beta_1R_2\vartheta_1\vartheta_0n\left(n+\frac{1}{2}\right)+8n^2\left((\alpha_1-\mu_1)n+\frac{\alpha_1}{2}-\frac{\theta_1}{2}\right)\vartheta_0^3\vartheta_1$
$-4n^2\left(n+\frac{1}{2}\right)(3\kappa^2\beta_1+2\kappa a_1+\rho+\lambda_1)\vartheta_1\vartheta_0=0,$

$\left(\frac{F'(\xi)}{F(\xi)}\right)^0$: $-2\beta_1(-4R_4\vartheta_1^2R_0+\vartheta_1^2R_2^2)(n-1)\left(n+\frac{1}{2}\right)+2n^2\left((\alpha_1-\mu_1)n+\frac{\alpha_1}{2}-\frac{\theta_1}{2}\right)\vartheta_0^4-2n^2\left(n+\frac{1}{2}\right)(3\kappa^2\beta_1+2\kappa a_1+\rho+\lambda_1)\vartheta_0^2=0.$

On solving the above algebraic equations $\left(\frac{F'(\xi)}{F(\xi)}\right)^0-\left(\frac{F'(\xi)}{F(\xi)}\right)^4$ using Maple, we have the result:

$R_4=R_4,\ R_0=R_0,\ n=1,\ \rho=-3\kappa^2\beta_1-2\beta_1R_2-2\kappa a_1-\lambda_1\ ,\vartheta_0=0,\ \vartheta_1=\sqrt{-\frac{6\beta_1}{3\alpha_1-2\mu_1-\theta_1}},$
(26)

provided $\beta_1(3\alpha_1-2\mu_1-\theta_1)<0.$
Substituting (26) into (23), we have the general solution of equation (22):

$$V(\xi)=\sqrt{-\frac{6\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\left[\frac{F'(\xi)}{F(\xi)}\right]. \qquad (27)$$

Now, according to Table A., Table B. and the general solution (27), we deduce the cases of soliton wave solutions of equations (1), (2) as follows:

Case-1. When $R_4=l^2,\ R_2=-(l^2+1),\ R_0=1,$ with $0<l<1$, $F(\xi)=sn(\xi,l),$ we derive the Jacobi-elliptic solutions

$$\varphi(x,t)=\left\{\sqrt{-\frac{6\beta_1}{3\alpha_1-2\mu_1-\theta_1}}\left(\frac{cn(\xi,l)dn(\xi,l)}{sn(\xi,l)}\right)\right\}e^{i[Q(x,t)+\sigma W(t)-\sigma^2t]}, \qquad (28)$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{29}$$

Specifically, as $l \to 1$, Equations (1), (2) exhibit straddled solitons

$$\varphi(x,t) = \left\{\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} sech(\xi) csch(\xi)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{30}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{31}$$

Case-2. When $R_4 = l^2$, $R_2 = -(1 + l^2)$, $R_0 = 1$, with $0 < l < 1$, $F(\xi) = cd(\xi, l)$, we derive the Jacobi-elliptic solutions

$$\varphi(x,t) = \left\{-\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} \left(\frac{(1 - l^2) sn(\xi, l)}{dn(\xi, l)^2 cd(\xi, l)}\right)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{32}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{33}$$

Case-3. When $R_4 = -l^2$, $R_2 = 2l^2 - 1$, $R_0 = 1 - l^2$, with $0 < l < 1$, $F(\xi) = cn(\xi, l)$, we derive the Jacobi-elliptic solutions

$$\varphi(x,t) = \left\{-\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} \left(\frac{dn(\xi, l)\, sn(\xi, l)}{cn(\xi, l)}\right)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{34}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{35}$$

Specifically, as $l \to 1$, Equations (1) and (2) exhibit dark solitons

$$\varphi(x,t) = \left\{-\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} tanh(\xi)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{36}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{37}$$

Figure 1: illustrates the simulations of soliton solution (36) in two-dimensional and three-dimensional plots with values: $\beta_1 = -1$, $\alpha_1 = 2$, $\mu_1 = 2$, $\theta_1 = \kappa = \omega = \rho = 1$, $\sigma = 0.02$ and $W_1(t) = \sqrt{t}$.

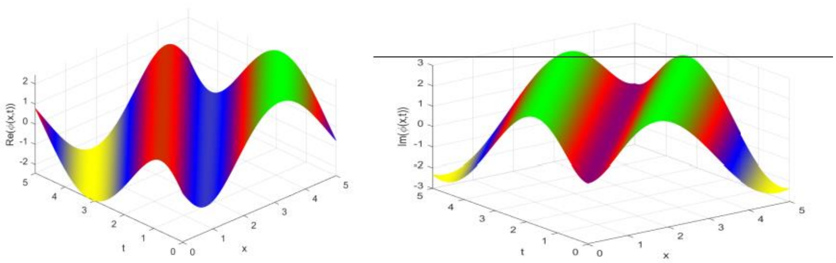

Figure 1. The profile of the dark soliton solution (36).

Case-4. When $R_4 = 1 - l^2$, $R_2 = 2 - l^2$, $R_0 = 1$, with $0 < l < 1$, $F(\xi) = sc(\xi, l)$, we derive the Jacobi-elliptic solutions

$$\varphi(x,t) = \left\{\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} \left(\frac{dn(\xi, l)}{cn(\xi, l)\, sn(\xi, l)}\right)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{38}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{39}$$

Specifically, as $l \to 1$, Equations (1) and (2) exhibit singular solitons

$$\varphi(x,t) = \left\{\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} coth(\xi)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{40}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{41}$$

Case-5. When $R_4 = \frac{1}{4}$, $R_2 = \frac{1-2l^2}{2}$, $R_0 = \frac{1}{4}$, with $0 < l < 1$, $F(\xi) = ns(\xi, l) \pm cs(\xi, l)$, we derive the Jacobi-elliptic solutions

$$\varphi(x,t) = \left\{\mp\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} ds(\xi, l)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{42}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \tag{43}$$

In particular, if $l \to 1$, then equations (1) and (2) have the singular solitons

$$\varphi(x,t) = \left\{\mp\sqrt{-\frac{6\beta_1}{3\alpha_1 - 2\mu_1 - \theta_1}} csch(\xi)\right\} e^{i[\vartheta(x,t)+\sigma W(t)-\sigma^2 t]}, \tag{44}$$

$$\psi(x,t) = \Omega_1 \varphi(x,t). \quad (45)$$

4. Conclusions

The generalized (F'/F) -expansion approach with the auxiliary equation $F'^2(\xi) = R_0 + R_2F^2(\xi) + R_4F^4(\xi)$ has been employed to obtain the solitons wave solutions of the stochastic R.K.L equation in magneto-optic waveguides with multiplicative white noise in the Itô sense and nonlinear power law. Dark, singular solitons solutions as well as Jacobi-elliptic function solutions are reported for the first time. Solitons solutions have been obtained by imposing specific constraints, which are also outlined in the current work. This study has been introduced a new model in the field of nonlinear optics, making the obtained results distinct from previously published works. We have presented a numerical simulation of the dark soliton (36), which represents the most important soliton solution, through two-dimensional and three-dimensional plots at small value of noise coefficient. In this figure, it is observed as the noise level increases, the surface becomes smoother after small transitional behaviors. This indicates that the presence of multiplicative noise affects the solutions and contributes to their stability. Ultimately, this study concludes that the noise effect, specifically the strength of the noise, has a significant impact on soliton solutions.

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