N-Cylindrical Fuzzy Interval Neutrosophic set

Fatma A. Hamad*

Basma M. Algamudi

Faculty of Arts and Science / Al-Marj, University of Benghazi, Libya

*fatmaadam40@yahoo.com

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Abstract:

In this paper, we define the notion of n-Cylindrical fuzzy interval neutrosophic sets as a combination of interval valued neutrosophic sets and n-Cylindrical fuzzy sets. Also, we give some definitions and algebraic operations on n-Cylindrical fuzzy interval neutrosophic sets sets. In addition, several related properties are also presented together with supporting proofs.

Keywords: Soft sets, Cylindrical fuzzy sets, fuzzy soft sets, neutrosophic sets, interval neutrosophic sets.

الدالة النونية التكرارية الضبابية لنتروفزك ذو الفترة المفتوحة

د. فاطمة آدم حامد

د. بسمة محمد القمودي

كلية الآداب والعلوم / المرج - جامعة بنغازي - ليبيا

الملخص:

في هذه الورقة العلمية قمنا بتعريف الدالة النونية التكرارية الضبابية لنتروفزك ذو الفترة؛ هذه الدالة عبارة عن تراكيب متعددة من دالة النتروفزك المفتوحة والدالة النونية التكرارية الضبابية. وكذلك أعطينا بعض التعريفات والعمليات الجبرية المتعلقة بالدالة النونية التكرارية الضبابية لنتروفزك ذو الفترة المفتوحة. بالإضافة إلى دراسة عدة مسلمات.

الكلمات المفتاحية: الدالة ذات الفترة، الدالة الضبابية التكرارية، الدالة الضبابية الناعمة، دالة نترو فزك، دالة نتروفزك ذات الفترة.

1. Introduction:

Many fields deal with the uncertain data may not be successfully modeled by the classical mathematics, since concept of uncertainty is too complicate and not clearly defined object. But they can be modeled on a number of different approaches including the probability theory, fuzzy set theory (Zadeh, 1965), rough set theory (Pawlak, 1982), neutrosophic set theory (Smarandache, 2005) and some other mathematical tools. This theory has been applied in many real applications to handle uncertainty. In 1999, Molodtsov (Molodtsov, 1999) successfully proposed a completely new theory called soft set theory by using classical sets because it has been pointed out that soft sets are not appropriate to deal with uncertain and fuzzy parameters.

Recently, researchers have shown an interest on research and application of a soft set. Different sets were rapidly developed and proposed in the literature such as (e.g. (Acar et al., 2010; Akta et al., 2007; Ayg¨unoglu & Aygun, 2009; Zhan & Jun, 2010)), ontology (e.g.(Jiang et al., 2010)), optimization (e.g.(Kovkov et al., 2007)), lattice (e.g.(Karaaslan et al., 2012; Nagarajan & Meenambigai, 2011) topology (e.g.(Cagman, 2011). Intuitionistic fuzzy sets can only handle incomplete information because the sum of degree true, indeterminacy and false is one in intuitionistic fuzzy sets. But neutrosophic sets can handle the indeterminate information and inconsistent information which exists commonly in belief systems in neutrosophic set since indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. It is mentioned in (Wang et al., 2005). Therefore, Maji firstly proposed neutrosophic soft sets with operations, which is free of the difficulties mentioned above, in (Maji, 2013). He also, applied to decision making problems in (Maji, 2013). After Maji, the studies on the neutrosophic soft set theory have been studied increasingly (e.g.(Abu Qamar & Hassan, 2019; Al-Quran & Hassan, 2017). In line with these developments, the purpose of this paper is to define n-Cylindrical fuzzy interval neutrosophic sets as the largest extension of fuzzy sets to deal with the problems involved periodicity information and varies with time in interval forms. The basic operations involving union, complement, intersection for n-Cylindrical fuzzy interval neutrosophic set was well defined. Subsequently, the basic properties of these operations related to n-Cylindrical fuzzy interval neutrosophic sets were given and mathematically proven. Finally, some examples are presented.

2. Preliminaries:

In this section we recall the definition and basic operations on fuzzy set and neutrosophic set. Throughout this paper, U denotes the universe of discourse.

Definition 2.1 (Zadeh, 1965) A fuzzy set A in U is defined by membership function $\mu: X \rightarrow [0,1]$whose membership value $\mu_A(x)$ shows the degree to which $x \in \mathrm{U}$ includes in the fuzzy set A for all $x \in \mathrm{U}$.

Definition 2.2 (Smarandache, 1999). A neutrosophic set A on U is $A = \{T_A(x), I_A(x), F_A(x) >; x \in U\}$ where $T_A(x), I_A(x), F_A(x): A \to]-0, 1^+[$ and -0 < $T_A(x), I_A(x), F_A(x)$< $n3^+$.

Definition 2.3 (Wang et al., 2005). An interval value neutrosophic set (IVN-sets) A in U is characterized by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $x \in U$; $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

Definition 2.4 (Arokiarani et al, 2013). A fuzzy neutrosophic set A on U is $A = \{T_A(x), I_A(x), F_A(x); x \in U\}$, where $T_A(x), I_A(x), F_A(x): A \to [0,1]$ and 0 < $T_A(x), I_A(x), F_A(x)$< 3.

Definition 2.5 (Kumari, 2022) An n- Cylindrical fuzzy neutrosophic A on U is an object of the form $A = \{x, \alpha_A(x), \beta_A(x), \gamma_A(x): x \in U\}$ where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$, called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition, (for all $x \in U$) ($0 \le \beta_A(x) \le 1$ and $0 \le \alpha_A n(x) + \gamma_A n(x) \le 1, n > 1$, is an integer. Here T and F are dependent neutrosophic components and I is 100% independent.

3. n-Cylindrical Fuzzy Interval Neutrosophic set:

With some examples and results, we give the concept of n- Cylindrical fuzzy interval neutrosophic set.

Definition 3.1. An n- Cylindrical fuzzy interval neutrosophic A on U is an object of the form $A =< x, [\alpha_A(x)^L, \alpha_A(x)^U], [\beta_A(x)^L, \beta_A(x)^U], [\gamma_A(x)^L, \gamma_A(x)^U] : x \in U >$ where ${\alpha_A}^{LU}(x) \in [0,1]$ called the degree of positive membership of x in A, ${\beta_A}^{LU}(x) \in [0,1]$, called the degree of neutral membership of x in A and ${\gamma_A}^{LU}(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition, (for all $x \in U$) , $0 \le \beta_A(x)^L, \beta_A(x)^U \le 1$ and $0 \le \alpha_A n(x)^L + \gamma_A n(x)^L \le 1$, $0 \le \alpha_A n(x)^U + \gamma_A n(x)^U \le 3, n > 1$, is an integer. Here T and F are dependent interval neutrosophic components and I is 100% independent.

For the convenience, $[\alpha_A(x)^L, \alpha_A(x)^U], [\beta_A(x)^L, \beta_A(x)^U], [\gamma_A(x)^L, \gamma_A(x)^U]$ is called as n-Cylindrical fuzzy interval neutrosophic number (n-CyFINN) and is denoted as $A =< {\alpha_A}^{LU}, {\beta_A}^{LU}, {\gamma_A}^{LU} >$.

Example 3.1 Suppose, $X = \{x_1, x_2, x_3\}$. The strength is x_1, the trust is x_2 and the price is x_3. The x_1, x_2 and x_3 values are given in [0,1]. They're obtained from some domain experts ' questionnaire, their choice could be degree of goodness, degree of indeterminacy and degree of poorness. A and B are the n-Cylindrical interval neutrosophic sets of X define by

$A =$
$< x_1, ([0.2,0.4],[0.3,0.5],[0.3,0.5]), x_2, ([0.5,0.7],[0,0.2],[0.2,0.3]), x_3, ([0.6,0.8],[0.2,0.3],[0.2,0.3]) >$, and $B =<$
$x_1, ([0.5,0.7],[0.1,0.3],[0.1,0.3]), x_2, ([0.2,0.3],[0.2,0.4],[0.5,0.8]), x_3, ([0.4,0.6],[0,0.1],[0.3,0.4]) >$.

Definition 3.2 Height of an n- CyFINS A is denoted as H(A) and is defined as $H(A) = [\max\{ \sup\beta_A(x)^L\}, \max\{\sup\beta_A(x)^U\}]: x \in U\}$. Thus height of an element $x \in U$ is H(x) and is equal to the degree of super neutral membership of x in U.

Example 3.2. From example(3.1) height of an n- CyFINS A is
$H(A) =< [\max\{\sup(0.3,0,0.2), \max\{\sup(0.5,0.2,0.3)\}] >= [0.3,0.5]$.

Definition 3.3. Let A an n- CyFINS then the peak of A is defined as
$\rho(A)=< [\max\{\alpha_A(x)^L, \beta_A(x)^L, \gamma_A(x)^L\}, \max\{\alpha_A(x)^U, \beta_A(x)^U \gamma_A(x)^U\}], x \in A >$, peak of an n-CyFINS can be the highest membership value of either positive, neutral or negative degree of membership. But height of an n-CyFINS is the highest value of degree of indeterminacy or degree of neutral membership.

Example 3.3. From example(3.1), peak of B is
$\rho_B(x_1) = [\max\{0,5.0.1,0.1\}, \max\{0.7,0.3,0.3\}] = [0.5,0.7]$.
$\rho_B(x_2) = [\max\{0.2,0.2,0.5\} \max\{0.3,0.4,0.8\}] = [0.5,0.8]$.
$\rho_B(x_3) = [\max\{0.4,0,0.3\} \max\{0.6,0.1,0.4\}] = [0,4,0.6]$.
Then $\rho(B) = [\max\{0.5,0.5,0.4\}, \max\{0.7,0.8,0.6\}] = [0.5,0.8]$.

Definition 3.4. Let A an n- CyFINS then A is called n-Right cylindrical fuzzy interval neutrosophic set (n-RCyFINS) if $H(A) = \rho(A)$.

4. The Basic Connectives:

Definition 4.1. Inclusion: For every two A, B ∈ n- CyFINS

i- $A \subseteq B$ iff for all $x \in U$
$\alpha_A(x)^L \leq \alpha_B(x)^L, \alpha_A(x)^U \leq \alpha_B(x)^U$ and $\beta_A(x)^L, \beta_B(x)^L, \beta_A(x)^U \leq \beta_B(x)^U$ and $\gamma_A(x)^L \geq \gamma_B(x)^L, \gamma_A(x)^U \geq \gamma_B(x)^U$.

ii- $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

Definition 4.2. The union of two n-CyFNISs A and B is

$A \cup B$
$= \{x, [\max(\alpha_A(x)^L, \alpha_B(x)^L), \max((\alpha_A(x)^U, \alpha_B(x)^U)], [\max(\beta_A(x)^L, \beta_B(x)^L, \max(\beta_A(x)^U, \beta_B(x)^U)], [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]: x \in U\}$.

Example 4.1. From example (3.1) the union of two n-CyFNISs A and B is

$A \cup B =< x_{1,}([0.5,0.7],[0.3,0.5],[0.1,0.3]), x_2,([0.5,0.7],[0.2,0.4],[0.2,0.3]), x_3,([0.6,0.8],[0.2,0.3],[0.2,0.3]) >.$

Definition 4.3. The intersection of two n-CyFNISs A and B is $A \cap B = \{x, [\min(\alpha_A(x)^L, \alpha_B(x)^L), \min((\alpha_A(x)^U, \alpha_B(x)^U)], [\min(\beta_A(x)^L, \beta_B(x)^L, \min(\beta_A(x)^U, \beta_B(x)^U)], [\max(\gamma_A(x)^L, \gamma_B(x)^L), \max(\gamma_A(x)^U, \gamma_B(x)^U)]: x \in U\}$.

Example 4.2. From example(3.1) intersection of two n-CyFNISs A and B is
$A \cap B = \{x_{1,} [\min(0.2,0.5), \min(0.4,0.7)],$
$[\min(0.3,0.1), \min(0.5,0.3)], [\max(0.3,0.1), \max(0.5,0.3)], x_{2,}, [\min(0.5,0.2), \min(0.7,0.3)]$
$[\min(0,0.2), \min(0.2,0.4)]\ \max(0.2,0.5)\ \max(0.3,0.8)], x_{3,}, [\min(0.6,0.4), \min(0.8,0.6)],$
$[\min(0.2,0), \min(0.3,0.1)], [\max(0.2,0.3), \max(0.3,0.4)]\},$
$=< x_{1,}[0.2,0.4],[0.1,0.3],[0.3,0.5], x_2,[0.2,0.3],[0,0.2], [0.5,0.8], x_{3,}[0.4,0.6], [0,0.1],[0.3,0.4] >.$

Definition 4.4. The complement of an n-CyFINS A is
$[\alpha_{\bar{A}}(x)^L, \alpha_{\bar{A}}(x)^U] = [\gamma_A(x)^L, \gamma_A(x)^U]$, $\beta_{\bar{A}}(x)^L=1-\beta_A(x)^U$, $\beta_{\bar{A}}(x)^U=1-\beta_A(x)^L$ and
$[\gamma_{\bar{A}}(x)^L, \gamma_{\bar{A}}(x)^U] = [\alpha_A(x)^L, \alpha_A(x)^U]$.

Example 4.3. From example(3.1) the complement of an n-CyFINS A is
$A = < x_{1,}([0.3,0.5],[0.3,0.2],[0.2,0.4]), x_2,([0.2,0.3],[0.8,1],[0.5,0.7]), x_3,([0.2,0.3], [0.7,0.8],[0.6,0.8]) >.$

Definition 4.5. The sum of two n-CyFINSs A and B is
$A \oplus B =< x, \left[\frac{\alpha_A(x)^L.\alpha_B(x)^L}{\alpha_A(x)^L+\alpha_B(x)^L}, \frac{\alpha_A(x)^U.\alpha_B(x)^U}{\alpha_A(x)^U+\alpha_B(x)^U}\right], [\max(\beta_A(x)^L, \beta_B(x)^L), \max(\beta_A(x)^U, \beta_B(x)^U)], [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]: x \in U >.$

Example 4.4. Assume that two A and B are n-CyFINSs defined as follows
$A = < x_{1,}([0.2,0.5],[0.1,0.5],[0.3,0.5]), x_2,([0.5,0.7],[0.1,0.3],[0.2,0]), x_3,([0,0.8],[0.1,0.3], [0.2,0.3]) >,$ and $B =< x_1,([0.5,0.7],[0.1,0.1],[0.1,0]), x_2,([0.2,0.4],[0,0.4],[0.3,0.8]), x_3, ([0.4,0.4],[0.1,0.1],[0.4,0.3]) >.$ Then the sum of A and B is
$A \oplus B =< x_{1,}, \left[\frac{01}{07}, \frac{035}{1.2}\right],$
$[0.1, 0.5],[0.1,0], x_2, \left[\frac{01}{07}, \frac{028}{1.1}\right], [0.1,0.4],[0.2,0], x_3, \left[0, \frac{032}{1.2}\right],$
$[0.1,0.3],[0.2,0.3] >.$

Definition 4.6. The difference of two n-CyFINSs A and B is

$A \ominus B =< x, [\max(\alpha_A(x)^L, \alpha_B(x)^L), \max(\alpha_A(x)^U, \alpha_B(x)^U)], [\min(\beta_A(x)^L, \beta_B(x)^L), \min(\beta_A(x)^U, \beta_B(x)^U)],$
$\left[\frac{\gamma_A(x)^L.\gamma_B(x)^L}{\gamma_A(x)^L+\gamma_B(x)^L}, \frac{\gamma_A(x)^U.\gamma_B(x)^U}{\gamma_A(x)^U+\gamma_B(x)^U}\right] : x \in U >.$

Definition 4.7. The product of two n-CyFINSs A and B is $A \otimes B = \{x, [\alpha_A(x)^L.\alpha_B(x)^L, \alpha_A(x)^U.\alpha_B(x)^U],$
$[\beta_A(x)^L.\beta_B(x)^L, \beta_A(x)^U.\beta_B(x)^U][\gamma_A(x)^L.\gamma_B(x)^L, \gamma_A(x)^U.\gamma_B(x)^U] : x \in U\}.$

Example 4.5. From example (4.4), the product of A and B is
$A \otimes B =$
$< x_{1,}([0.1,0.35], [0.01,0.05], [0.03,0]), x_2, ([0.1,0.28], [0,0.12], [0.06,0]), x_3, ([0,0.32], [0.01,0.03], [0.08,0.12]) >.$

Definition 4.8. The division of two n-CyFINSs A and B is
$A \oslash B =$
$\{x, [\min(\alpha_A(x)^L, \alpha_B(x))^L, \min(\alpha_A(x)^U, \alpha_B(x)^U)], [\beta_A(x)^L.\beta_B(x)^L$
$\beta_A(x)^U.\beta_B(x)^U], [\max(\gamma_A(x)^L.\gamma_B(x)^L), \max(\gamma_A(x)^U.\gamma_B(x)^U)].$

Example 4.5. From example (4.4), the division t of A and B is
$A \oslash B =$
$< x_{1,}([0.2,0.5], [0.01,0.05], [0.3,0.5]), x_2, ([0.2,0.4], [0,0.12], [0.3,0.8]), x_3, ([0,0.4], [0.01,0.03], [0.4,0.3] >.$

5. Operations n-cylindrical fuzzy interval neutrosophic set:

Some properties of the defined operations on n-CyFINSs are considered.

Proposition 5.1.

The following relations (mostly equalities) are valid for every three n-CyFINSs A, B and C:

a) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

b) $A \cup B = B \cup A$ & $A \cap B = B \cap A$.

c) $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$.

d) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

e) $A \cap A = A$ & $A \cup A = A$.

f) De Morgan's Law for A& B i.e., $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

g) $A \oplus B = B \oplus A$.

h) $A \otimes B = B \otimes A$.

Proof: (a)From Definition (4.1), $A \subseteq B$ means
$(\forall x \in U, \{[\alpha_A(x)^L \leq \alpha_B(x)^L, \alpha_A(x)^U \leq \alpha_B(x)^U], [\beta_A(x)^L \leq \beta_B(x)^L, \beta_A(x)^U \leq \beta_B(x)^U]$ and

$[\gamma_A(x)^L \geq \gamma_B(x)^L, \gamma_A(x)^U \geq \gamma_B(x)^U]$. And $B \subseteq C$ implies that $(\forall x \in U$, $[\alpha_B(x)^L \leq \alpha_C(x)^L, \alpha_B(x)^U \leq \alpha_C(x)^U]$,
$[\beta_B(x)^L \leq \beta_C(x)^L, \beta_B(x)^U \leq \beta_C(x)^U]$ and $[\gamma_B(x)^L \geq \gamma_C(x)^L, \gamma_B(x)^U \geq \gamma_C(x)^U]$ i.e., $\forall x \in U$, $[\alpha_A(x)^L \leq \alpha_B(x)^L \leq \alpha_c(x)^L, \alpha_A(x)^U \leq \alpha_B(x)^U \leq \alpha_C(x)^U]$, $[\beta_A(x)^L \leq \beta_B(x)^L \leq \beta_C(x)^L, \beta_A(x)^U \leq \beta_B(x)^U \leq \beta_C(x)^U]$
and $[\gamma_A(x)^L \geq \gamma_B(x)^L \geq \gamma_C(x)^L, \gamma_A(x)^U \geq \gamma_B(x)^U \gamma_C(x)^U]$ Thus, $A \subseteq C$.

b) From Definition (4.4), we have $A \cup B = \{x, [\max(\alpha_A(x)^L, \alpha_B(x)^L), \max((\alpha_A(x)^U, \alpha_B(x)^U)], [\max(\beta_A(x)^L, \beta_B(x)^L), \max(\beta_A(x)^U, \beta_B(x)^U)], [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]: x \in U\}$,
$= \{x, [\max(\alpha_B(x)^L, \alpha_A(x)^L), \max((\alpha_B(x)^U, \alpha_A(x)^U)], [\max(\beta_B(x)^L, \beta_A(x)^L), \max(\beta_B(x)^U, \beta_A(x)^U)], [\min(\gamma_B(x)^L, \gamma_A(x)^L), \min(\gamma_B(x)^U, \gamma_A(x)^U)]: x \in U\}$,
$= B \cup A$. The proof of part $A \cap B = B \cap A$ is similar to part (b).

d) By using Definition (4.4) and (4.5), we have
$(A \cup B) \cap C = \{x, [\min(\alpha_C(x)^L, (\max(\alpha_A(x)^L, \alpha_B(x)^L)), \min((\alpha_C(x)^U, \max((\alpha_A(x)^U, \alpha_B(x)^U))], [\min(\beta_C(x)^L, (\max(\beta_A(x)^L, \beta_B(x)^L)), \min(\beta_C(x)^U, \max(\beta_A(x)^U, \beta_B(x)^U))], [\max(\gamma_C(x)^L (\min(\gamma_A(x)^L, \gamma_B(x)^L), \max(\gamma_A(x)^U (\min(\gamma_A(x)^U, \gamma_B(x)^U))])\}$,
$= \{x, [\max(\min(\alpha_C(x)^L, \alpha_A(x)^L), \min(\alpha_C(x)^L, \alpha_B(x)^L)), \max(\min(\alpha_C(x)^U, \alpha_A(x)^U), \min(\alpha_C(x)^U,$

$\alpha_B(x)^U)][\max(\min(\beta_C(x)^L, (\beta_A(x)^L), \min(\beta_C(x)^L, \beta_B(x)^L), (\max(\min(\beta_C(x)^U, \beta_A(x)^U), \min(\beta_c(x)^U, \beta_A(x)^U)], [\min(\max(\gamma_C(x)^L, \gamma_A(x)^L), \max(\gamma_C(x)^L, \gamma_B(x)^L)), \min(\max(\gamma_C(x)^U, \gamma_A(x)^U), \max(\gamma_C(x)^U, \gamma_B(x)^U))]$, therefore $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

In a similar way, we can prove $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

e) The proof is straightforward.

f) The proof of part (1), by using Definition (4.4) and (4.2) $(A \cup B)^c = \{x, [\max(\alpha_A(x)^L, \alpha_B(x)^L), \max((\alpha_A(x)^U, \alpha_B(x)^U)]^c, [\max(\beta_A(x)^L, \beta_B(x)^L, \max(\beta_A(x)^U, \beta_B(x)^U)]^c, [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]^c : x \in U\}$, thus
$= \{x, [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min((\gamma_A(x)^U, \gamma_B(x)^U)], [\min(1 - \beta_A(x)^U, 1 - \beta_B(x)^U,$

$\min(1-\beta_A(x)^L, 1 - \beta_B(x)^L)], [\min(\alpha_A(x)^L, \alpha_B(x)^L), \min(\alpha_A(x)^U, \alpha_B(x)^U)] : x \in U\}$

$=\{x, [\gamma_A(x)^L, \gamma_A(x)^U], [1-\beta_A(x)^U, 1-\beta_A(x)^L], [\alpha_A(x)^L, \alpha_A(x)^U]\} \cap \{x, [\gamma_B(x)^L, \gamma_B(x)^U]$

$[1-\beta_B(x)^U, 1-\beta_B(x)^L], [\alpha_B(x)^L, \alpha_B(x)^U]\}$,

$=A^c \cap B^c$, therefore $(A\cup B)^c = A^c \cap B^c$.

f) The proof of part (2), by using Definition (4.4) and (4.3) we have $(A\cup B)^c = \{x, [\min(\alpha_A(x)^L, \alpha_B(x)^L), \min((\alpha_A(x)^U, \alpha_B(x)^U)]^c,$ $[\min(\beta_A(x)^L, \beta_B(x)^L, \min(\beta_A(x)^U, \beta_B(x)^U)]^c, [\max(\gamma_A(x)^L, \gamma_B(x)^L), \max(\gamma_A(x)^U, \gamma_B(x)^U)]^c : x \in U\}$, thus

$= \{x, [\max(\gamma_A(x)^L, \gamma_B(x)^L), \max((\gamma_A(x)^U, \gamma_B(x)^U)], [\max(1-\beta_A(x)^U, 1 - \beta_B(x)^U,$

$\max(1-\beta_A(x)^L, 1 - \beta_B(x)^L)], [\min(\alpha_A(x)^L, \alpha_B(x)^L), \min(\alpha_A(x)^U, \alpha_B(x)^U)] : x \in U\}$

$=\{x, [\gamma_A(x)^L, \gamma_A(x)^U], [1-\beta_A(x)^U, 1-\beta_A(x)^L], [\alpha_A(x)^L, \alpha_A(x)^U]\} \cup \{x, [\gamma_B(x)^L, \gamma_B(x)^U]$

$[1-\beta_B(x)^U, 1-\beta_B(x)^L], [\alpha_B(x)^L, \alpha_B(x)^U]\}$,

$=A^c \cup B^c$, thus $(A \cap B)^c = A^c \cup B^c$.

g) From Definition (4.5) we have $A \oplus B =< x, \left[\frac{\alpha_A(x)^L.\alpha_B(x)^L}{\alpha_A(x)^L+\alpha_B(x)^L}, \frac{\alpha_A(x)^U.\alpha_B(x)^U}{\alpha_A(x)^U+\alpha_B(x)^U}\right],$

$[\max(\beta_A(x)^L, \beta_B(x)^L), \max(\beta_A(x)^U, \beta_B(x)^U)], [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)] >,$

$=< x, \left[\frac{\alpha_B(x)^L.\alpha_A(x)^L}{\alpha_B(x)^L+\alpha_A(x)^L}, \frac{\alpha_B(x)^U.\alpha_A(x)^U}{\alpha_B(x)^U+\alpha_A(x)^U}\right],$ $[\max(\beta_B(x)^L, \beta_A(x)^L), \max(\beta_B(x)^U, \beta_A(x)^U)],$

$[\min(\gamma_B(x)^L, \gamma_A(x)^L), \min(\gamma_B(x)^U, \gamma_A(x)^U)] > = B \oplus A$.

h) $A \otimes B = \{x, [\alpha_A(x)^L. \alpha_B(x)^L, \alpha_A(x)^U, \alpha_B(x)^U], [\beta_A(x)^L. \beta_B(x)^L, \beta_A(x)^U. \beta_B(x)^U]$ $[\gamma_A(x)^L. \gamma_B(x)^L, \gamma_A(x)^U. \gamma_B(x)^U],$

$= \{x, [\alpha_B(x)^L. \alpha_A(x)^L, \alpha_B(x)^U, \alpha_A(x)^U], [\beta_B(x)^L. \beta_A(x)^L, \beta_B(x)^U. \beta_A(x)^U]$

$[\gamma_B(x)^L. \gamma_A(x)^L, \gamma_B(x)^U. \gamma_A(x)^U]\} = B \otimes A$.

Definition 5.1. Let U_1 and U_2 be two universes and $A =< x, [\alpha_A(x)^L, \alpha_A(x)^U], [\beta_A(x),$ $, \beta_A(x)^U], [\gamma_A(x)^L, \gamma_A(x)^U] : x \in U_1 >$ and $B =< x, [\alpha_B(x)^L, \alpha_B(x)^U], [\beta_B(x)^L$

$, \beta_B(x)^U], [\gamma_B(x)^L, \gamma_B(x)^U] : x \in U_2 >$ two n- CyFINSs. The product of these two n-CyFINSs is defined as follows $A \times B = \{x, [\alpha_A(x)^L + \alpha_B(x)^L - \alpha_A(x)^L. \alpha_B(x)^L, \alpha_A(x)^U + \alpha_B(x)^U - \alpha_A(x)^U. \alpha_B(x)^U],$
$[\beta_A(x)^L. \beta_B(x)^L , \beta_A(x)^U. \beta_B(x)^U], [\gamma_A(x)^L. \gamma_B(x)^L, \gamma_A(x)^U. \gamma_B(x)^U]\}.$

Example 5.1. Let $U_1 = \{x_1, x_2\}$ and $U_2 = \{y_1, y_2\}$ be two universes and
$A \subseteq U_1 =< x_1, ([0.2,0.4], [0.3,0.5], [0.3,0.5]), x_2, ([0.5,0.7], [0,0.2], [0.2,0.3]) >,$

$B \subseteq U_2 =< y_1, ([0.5,0.7], [0.1,0.3], [0.1,0.3]), y_2, ([0.2,0.3], [0.2,0.4], [0.5,0.8]) >.$

Then $A \times B =< \big((x_1, y_1), [0.2 + 0.5 - 0.2 \times 0.5, 0.4 + 0.7 - 0.4 \times 0.7][0.3 \times 0.1, 0.5 \times 0.3], [0.3 \times 0.1, 0.5 \times 0.3]\big), (((x_2, y_2), [0.5 + 0.2 - 0.5 \times 0.2, 0.7 + 0.3 - 0.7 \times 0.3], [0 \times 0.2, 0.2 \times 0.4], [0.2 \times 0.5, 0.3 \times 0.8]) >,$

$=< \big((x_1, y_1), [0.6, 0.82], [0.03,0.15], [0.03,0.15]\big), ((x_2, y_2), [0.6,0.79], [0,0.08], [0.1,0.24]) > .$

Preposition: 5.2.

a) $A \times B = B \times A$ iff $A = B$.
b) $(A \times B) \times C = A \times (B \times C)$.
c) $(A \cup B) \times C = (A \times C) \cup (B \times C)$.
d) $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

Proof.

b) $(A \times B) \times C = \{(x, [\alpha_A(x)^L + \alpha_B(x)^L - \alpha_A(x)^L. \alpha_B(x)^L, \alpha_A(x)^U + \alpha_B(x)^U - \alpha_A(x)^U. \alpha_B(x)^U],$
$[\beta_A(x)^L. \beta_B(x)^L , \beta_A(x)^U. \beta_B(x)^U], [\gamma_A(x)^L. \gamma_B(x)^L, \gamma_A(x)^U. \gamma_B(x)^U]) \times< x, [\alpha_C(x)^L, \alpha_C(x)^U], [\beta_C(x), \beta_C(x)^U], [\gamma_C(x)^L, \gamma_C(x)^U] >,$

$=\{x, [(\alpha_A(x)^L + \alpha_B(x)^L) + \alpha_C(x)^L - (\alpha_A(x)^L. \alpha_B(x)^L). \alpha_C(x)^L, (\alpha_A(x)^U + \alpha_B(x)^U) + \beta_C(x)^U - (\alpha_A(x)^U. \alpha_B(x)^U). \alpha_C(x)^U], [(\beta_A(x)^L. \beta_B(x)^L). \beta_C(x)^L, (\beta_A(x)^U. \beta_B(x)^U). \beta_C(x)^U], [(\gamma_A(x)^L. \gamma_B(x)^L). \gamma_C(x)^L, (\gamma_A(x)^U. \gamma_B(x)^U). \gamma_C(x)^U]\},$

$=\{x, [\alpha_A(x)^L + (\alpha_B(x)^L + \alpha_C(x)^L) - \alpha_A(x)^L. (\alpha_B(x)^L. \alpha_C(x)^L), \alpha_A(x)^U + (\alpha_B(x)^U + \beta_C(x)^U) - \alpha_A(x)^U. (\alpha_B(x)^U. \alpha_C(x)^U)], [\beta_A(x)^L. (\beta_B(x)^L. \beta_C(x)^L), \beta_A(x)^U. (\beta_B(x)^U. \beta_C(x)^U)], [\gamma_A(x)^L. (\gamma_B(x)^L. \gamma_C(x)^L), \gamma_A(x)^U. (\gamma_B(x)^U. \gamma_C(x)^U)]\}$.Thus$= A \times (B \times C)$.

c) From Definition (4.2) we have $(A \cup B) \times C = \{x, [\max(\alpha_A(x)^L, \alpha_B(x)^L), \max((\alpha_A(x)^U, \alpha_B(x)^U)], [\max(\beta_A(x)^L, \beta_B(x)^L,$
$\max(\beta_A(x)^U, \beta_B(x)^U)], [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)] \times <$
$x, [\alpha_C(x)^L, \alpha_C(x)^U], [\beta_C(x),$
$\beta_C(x)^U], [\gamma_C(x)^L, \gamma_C(x)^U] >,$
$=\{(x, [\max(\alpha_A(x)^L + \alpha_C(x)^L - \alpha_A(x)^L . \alpha_C(x)^L), \max(\alpha_A(x)^U + \alpha_C(x)^U - \alpha_A(x)^U . \alpha_C(x)^U)],$
$[\max(\beta_A(x)^L . \beta_C(x)^L), \max(\beta_A(x)^U . \beta_C(x)^U)], [\min(\gamma_A(x)^L . \gamma_C(x)^L), \min(\gamma_A(x)^U . \gamma_C(x)^U])$
$)$
$([\max(\alpha_B(x)^L + \alpha_C(x)^L - \alpha_B(x)^L . \alpha_C(x)^L), \max(\alpha_B(x)^U + \alpha_C(x)^U - \alpha_B(x)^U . \alpha_C(x)^U)],$
$[\max(\beta_B(x)^L . \beta_C(x)^L), \max(\beta_B(x)^U . \beta_C(x)^U)], [\min(\gamma_B(x)^L . \gamma_C(x)^L), \min(\gamma_B(x)^U . \gamma_C(x)^U])$
$)\},$
$=\{(x, [\max((\alpha_A(x)^L + \alpha_C(x)^L - \alpha_A(x)^L . \alpha_C(x)^L), (\alpha_B(x)^L + \alpha_C(x)^L - \alpha_B(x)^L . \alpha_C(x)^L), \max(\alpha_A(x)^U + \alpha_C(x)^U - \alpha_A(x)^U . \alpha_C(x)^U, (\alpha_B(x)^U + \alpha_C(x)^U - \alpha_B(x)^U . \alpha_C(x)^U)], [\max((\beta_A(x)^L . \beta_C(x)^L, \beta_B(x)^L .$
$\beta_C(x)^L),$
$\max(\beta_A(x)^U . \beta_C(x)^U, \beta_B(x)^U . \beta_C(x)^U)], [\min(\gamma_A(x)^L . \gamma_C(x)^L, \gamma_B(x)^L . \gamma_C(x)^L),$
$\min(\gamma_A(x)^U .$
$\gamma_C(x)^U, \gamma_B(x)^U . \gamma_C(x)^U)]\}$. Hence $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

d) The proof is similar to that of part (c).

Conclusion:

In this paper, we introduced a new type of set called n-Cylindrical fuzzy interval neutrosophic sets. The basic algebraic operations on n-Cylindrical fuzzy interval neutrosophic sets namely sum, difference, product and division with illustrative examples were presented. Subsequently, the basic properties of these operations such as commutative law De Morgan's Law, and relevant laws are mathematically proven.

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