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The rational quadratic Bezier-Like curve with G1 and G2 continuity

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Abstract:

In this work, the quadratic Bezier-Like curve with two shape parameters is discussed. The rational quadratic Bezier-Like curve inherits all geometric properties of quadratic Bezier curve. The shape of the curve can be adjust by altering the values of the shape parameters that provides a control on the shape of the curve. Also in this paper, we finding new theorems of rational quadratic Bezier-Like curve to evaluate the minimum value of weights that guaranty the curve is not going out of its control polygon. The composition of two segment of rational quadratic Bezier-Like curve using C^1, C^2, G^1 and G^2 continuity are studied.

Key words: Quadratic Bezier-Like curve, Rational quadratic Bezier-Like curve, shape parameters, Parametric continuity, Geometric continuity.

المنحني الشبيه لمنحني بيزير التربيعي مع استمرارية G1,G2

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في هذا العمل تم دراسة المنحني التربيعي الشبيه لمنحني بيزير مع ضوابط التحكم بالشكل، منحني بيزير التربيعي الكسري يرث كل الخصائص الهندسية لمنحني بيزير الأصلي، شكل المنحني يمكن ان يتغير بواسطة تبديل عوامل تغير الشكل التي تزود التحكم في شكل المنحني.

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وفي هذه الورقة ايضا تم تعريف نظرية جديدة للمنحني الشبيه لمنحني بيزير الكسري لحساب اصغر قيمة للأوزان التي تضمن عدم خروج المنحني من مضلع التحكم . ايضا تم دراسة تركيب قطعتين من المنحني الشبيه لمنحني بيزير الكسري باستخدام **C1,C2,G1,G2** .

الكلمات المفتاحية: منحنى تربيعي يشبه Bezier، معلمات الشكل، استمرارية حدودية، استمرارية هندسية.

1. Introduction

Curve and surface design is an important subject and play significant role in computer aided geometric design (CAGD) and computer graphics (Hoscheck & Lasser, 1993). The parametric representation of curves and surfaces with shape parameter is most convenient for design and it has much attention of the designer (Bashir, Abbas, & Ali, 2013) (Bahir, Abbas, Awang, & Ali, 2012). In Geometric, modeling the control polygon is important tool for curve design. The curve being models tends to preserve the shape of its control polygon (Han, 2003). To cope the deficiency of Bezier curve some authors constructed new curves representation by introducing parameters into basis functions where the new curve is similar to the Bezier curve. These new curves have flexible shape and at the same time satisfy many basic properties as the Bezier curve. For more details, see Han (Han,, 2004), Yan and Liang **(**Yan & Liang, 2011). Also, even though Bezier curve is the powerful modeling tool in CAD. System there are authors who proposed new curve definition. Yan and Liang (Yan & Liang, 2011) gives an extension of the Bezier curve by constructing the Bezier-Like curve and rectangular Bezier-Like surface. The new curve and surface inherits many properties of Bezier curve and surface. The shape of the curve can be modified by changing the values of the shape parameters without altering the control polygon, many researchers have studied this fact. Bashir (Bashir, Abbas, & Ali, 2013) (Bahir, Abbas, Awang, & Ali, 2012) have described the quadratic trigonometric Bezier curve with single and two shape parameters respectively, these two curves are analogous to ordinary quadratic Bezier curve. Liu and Zahang (Liu & Zhang, 2011) have been focused on the study of TC-Bezier curve with shape parameter. Abbas (Abbas, Yahya, V, & Ali, 2012) have developed spur gear tooth design using T-Bezier function. The

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composition of two curve segments using C^1 ,G^2 continuity discussed by Bashir (Bahir, Abbas, Awang, & Ali, 2012) and Han (Han, 2003) introduced piecewise quadratic trigonometric polynomials curve with C^2 continuity. In this work we construct rational quadratic Bezier-Like curve with two shape parameters and discuss the geometric properties of this curve. There is no difference in structure between Bezier-Like curve and ordinary Bezier curve, so it is not difficult to adapt a Bezier-Like curve to CAD system that already use Bezier curve. we also define a new basis functions which are combination of quadratic and linear polynomial with two shape parameters. The present work is organized as follows. In section 2, the basic functions of quadratic Bezier-Like curve with two shape parameters are established and the properties of basis functions are shown. In section 3. The rational quadratic Bezier-Like curve with some geometric properties are discussed. In section 4. The shape control of rational quadratic Bezier-Like curve is presented. In section 5. Includes the conditions on the weight for the curve to pass through a fixed point and the curve does not going out of its control polygon. In section 6. Describe the conditions of continuity of two segment of proposes curve. In section 7. the conclusion is presented.

2. Qudratic Bezier-Like basis functions

We construct a quadratic Bezier-Like basis functions with two shape parameters as follows:

2.1 Definition

The following functions with two shape parameters, which are a combination of quadratic and linear polynomial, are defined as quadratic Bezier-Like basis functions.

$$\begin{aligned} f_0(\theta) &= (1-\gamma\theta)(1-\theta)^2 \\ f_1(\theta) &= \theta(\theta-1)(-2+\gamma(1+\theta)-\mu\theta) \\ f_2(\theta) &= (1-\mu(1-\theta))\theta^2 \end{aligned} \qquad (2.1)$$

Where $\theta\epsilon[0,1]$ *and* $\gamma, \mu\epsilon[-2,1]$.

2.2 The Properties of Basis Functions

The basis functions of quadratic Bezier-Like curve holds the following properties:

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- **Non-negative (Positivity):** $f_i\,(\theta) \geq 0,\ i = 0, 1, 2$
- **Partition of unity:** $\sum_{i=o}^{2} f_i\,(\theta) = 1$
- **Monotonicity:** $f_0(\theta)$ is monotonically decreasing and $f_2(\theta)$ is monotonically increasing for the specific value of the parameters γ and μ.
- **Symmetry:** $f_i(\theta;\gamma,\mu) = f_{2-i}(1-\theta;\mu,\gamma), i = 0,1,2.$

Figure 1 display the curves of the quadratic Bezier-Like basis functions for $\gamma = 1, \mu = -1$ (dashed lines), $\gamma = \mu = 0$ (dotted lines), $\gamma = 0.5, \mu = -0.5$ (solid lines) and for $\gamma = -1, \mu = 1$ (dotted dashed lines).

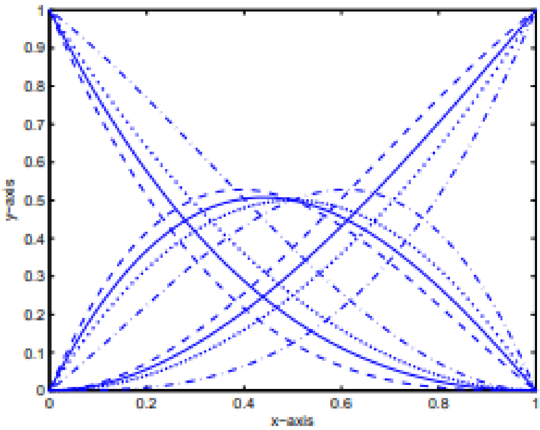


Figure 1: Rational quadratic Bezier-Like with γ = 1, μ = −1

3. The Rational Quadratic Bezier-Like Curve

In this section, we construct rational quadratic Bezier-Like Curve with two shape parameters and discuss some of their properties.

3.1 Definition

A rational quadratic Bezier-Like curve confine with the control points $(P_i, i = 0, 1, 2)$ and the weight v with two shape parameters γ and μ is defined as:

$$r(\theta) = \frac{f_0 P_0 + f_1 P_1\,v + f_2 P_2}{f_0 + f_1 v + f_2}, \theta\epsilon[0,1], \gamma, \mu\epsilon[-2, \quad 1]$$

and v must be positive. If $v = 1$ the rational quadratic Bezier-Like curve become non- rational Bezier-Like curve since the sum of basis functions f_0, f_1, f_2 equal to one.

3.2 The Properties of Rational Quadratic Bezier-Like Curve

A rational quadratic Bezier-Like curve satisfied the following properties:

- **End points property**

$$r(0) = P_0, r(1) = P_2$$
$$r'(0) = (2+\gamma)v(P_1 - P_0), r'(1) = (2+\mu)v(P_2 - P_1)$$
$$r''(0) = 2((1-\mu)(P_2 - P_0) - (2+\gamma(2+\gamma)+\mu)(P_0 - P_1)\boldsymbol{v} + (\boldsymbol{2}+\boldsymbol{\gamma})^{\boldsymbol{2}}(P_0 - P_1)v^2)$$
$$r''(1) = 2((1-\gamma)(P_0 - P_2) - (2+\mu(2+\mu)+\gamma)(P_2 - P_1)\,\boldsymbol{v} + (\boldsymbol{2}+\boldsymbol{\mu})^{\boldsymbol{2}}(P_2 - P_1)v^2)$$

- **Geometric invariance**

Invariance of the shape of Bezier curve is preserved by partition of unity property of
Bernstein polynomials under the influence of translation and rotation of its control points.
Geometric invariance satisfies the following conditions.

1. $r(\theta,\gamma,\mu,P_0 + s, P_1 + s, P_2 + s) = r(\theta,\gamma,\mu,P_0,P_1,P_2) + s$
2. $r(\theta,\gamma,\mu,P_0 * M, P_1 * M, P_2 * M) = r(\theta,\gamma,\mu,P_0,P_1,P_2) * M$

Where s is arbitrary vector in R^2, and M is arbitrary matrix 2×2.

- **Convex hull property**

The convex hull of Bezier-Like curve indicate to the convex hull of its control points.
The quadratic Bezier-Like curve defined by the three control points lie in the convex
hull.

- **Symmetry**

If the control points specified in the opposite order and interchange the shape parameters
γ, μ by using the symmetry property of basis functions then the same shape of Bezier-Like curve will be generated but in the opposite direction.

$r(\theta,\gamma,\mu,P_0,P_1,P_2) = r(1-\theta,\mu,\gamma,P_2,P_1,P_0), \theta\epsilon[0,1], \gamma,\mu\epsilon[-2,1].$

4. Constraints for shape parameters of rational quadratic Bezier-Like curve using endpoint curvatures

Suppose $P_i(t_i, s_i)$ be the control points of rational quadratic Bezier-Like curve and r_0, r_1
be the radii of curvature at $\theta = 0, \theta = 1$ respectively. By using the curvature of the curve
in Hoscheck (Hoscheck & Lasser, 1993) is defined as:

$$k(\theta) = \frac{r'(\theta) \times r''(\theta)}{||r'(\theta)||^3}$$

Such that $k(0) = \frac{1}{r_0}$ and $k(1) = \frac{1}{r_1}$, the value of γ and μ are:

$$\gamma = -2 + \frac{1}{v}\sqrt{\frac{2(-1+\mu)r_0 R_1}{R_2}}$$

$$\mu = -2 + \frac{1}{v}\sqrt{\frac{2(-1+\gamma)r_1 R_1}{R_3}}$$

Where

$R_1 = t_2(s_1 - s_0) + t_1(s_0 - s_2) + t_0(s_2 - s_1)$

$R_2 = ((t_1 - t_0)^2 + (s_1 - s_0)^2)^{3/2}$

$R_3 = ((t_2 - t_1)^2 + (s_2 - s_1)^2)^{3/2}$

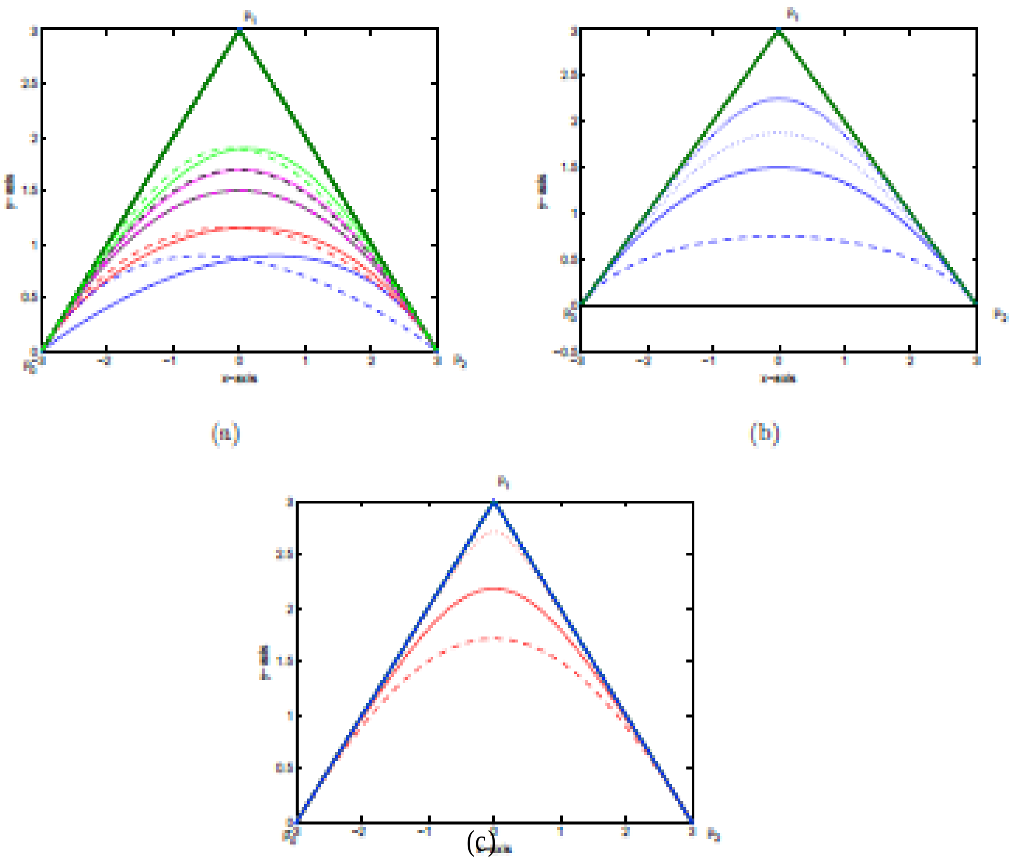
5. The Rational Quadratic Bezier-Like Curve with Shape Control

The shape parameters γ and μ control the shape of rational Bezier-Like curve and change the weight v is effected by different way from moving towards the control points. Suppose the control points are known, taking v and γ constants, the curve will move toward P_0P_1 such that the value of μ increasing in the range [−2, 1], also for constants values of v and μ the curve become closer to P1P2 such that the value of γ increasing in the same range, and if γ and μ increase together, with constant v, the curve will move near to the control polygon.

In Fig. 2(a) the curve sloping to the left side (moves toward the P_0P_1) (dashed lines),is generated by setting the values of μ as $\mu = -2$ (blue), $\mu = -1$

(red), $\mu = 0$ (pink),$\mu = 0.5$ (black), $\mu = 1$ (green) are shown, by the similar way the curve propensity to the right side (moves toward the P_1P_2) (solid lines) produced by placement of the values of $\gamma = -2$ (blue), $\gamma = -1$ (red), $\gamma = 0$ (black), $\gamma = 0.5$ (pink), $\gamma = 1$ (green) are presented. In Fig. $2(b)$, the curve is designed by changing the values of γ and μ in the same time from small to big values with constant v.

For $\gamma = \mu = -1$ (dashed line), $\gamma = \mu = 0$ (solid line), $\gamma = \mu = 0.5$ (dotted dotted line), $\gamma = \mu = 1$ (dotted line), $\gamma = \mu = -2$ (black). In Fig. $2(c)$, the curve is designed with fixed γ and μ and choosing the value of v as $v = 4$ (dashed line), $v = 8$ (solid line), v = 28 (dotted line), $v = 0$ (black), and for $v = 400$ generates the three control points.



(c)

Figure 2: The rational quadratic Bezier-Like curve with shape control

6. Some Results of Rational Quadratic Bezier-Like Curve

Theorem 1.
A rational quadratic Bezier-Like curve will be a straight line $if\, \gamma = \mu = -2$.
Proof. When $\gamma = \mu = -2$, the rational quadratic B´ezier-Like curve is given as:

$$r(\theta) = \frac{(1+2\theta)(1-\theta)^2 P_0 + (1+2(1-\theta))\theta^2 P_2}{(1+2\theta)(1-\theta)^2 + (1+2(1-\theta))\theta^2}$$

is a parametric equation of a straight line.
Theorem 2.
Suppose $P_i, i = 0,1,2$ be a control points of rational quadratic Bezier-Like curve. Let $P(t,s)$ be a point located on the median of triangle $\Delta P_0P_1P_2$ and lies on the
curve. Then the minimum value of weight v for which the curve is within its control
polygon is given as:

$$v_{min} = \frac{1-c}{3c}$$

where c is a constant.
Proof. Assume that the control points define as $P_0 = (t_0, s_0)$, $P_1 = (t_1, s_1)$, $P_2 = (t_2, s_2)$. Let T be a midpoint of P_0P_2 and let $P(t,s)$ be a point on the median of P_1T. Suppose P divides the median in the radio $c : 1 - c$. Then the coordinate of the point $P(t,s)$ are:

$$P((1-c)t_1 + \frac{c(t_0 - t_2)}{2}, (1-c)s_1 + \frac{c(s_0 - s_2)}{2})$$

Since the point P located in the median then the following relation is true:

$$(1-c)t_1 + \frac{c(t_0 - t_2)}{2} = \frac{f_0t_0 + f_1t_1v + f_2t_2}{f_0 + f_1v + f_2}$$

(6.1)

$$(1-c)s_1 + \frac{c(s_0 - s_2)}{2} = \frac{f_0s_0 + f_1s_1v + f_2s_2}{f_0 + f_1v + f_2}$$

Where

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$$f_0 = \frac{2-\gamma}{8}, f_1 = \frac{4+\gamma+\mu}{8}, f_2 = \frac{2-\mu}{8}$$

To find the values of γ and μ must be solved the equation (6.1) for γ, μ by solving the equation will have:

$$\gamma = \mu = -\frac{2(-1+c+cv)}{1+c(-1+v)} \qquad (6.2)$$

v is minimum when $\gamma = \mu = 1$. By putting $\gamma = \mu = 1$ in the equation (6.2) and rearrange it for v will get:

$$v_{min} = \frac{1-c}{3c}$$

6.1 **Numerical Example**

Suppose $P_i, i = 0, 1, 2$ are the control points of rational quadratic Bezier-Like curve. Figure 3 refer to the rational quadratic Bezier-Like curve design by $c = \frac{1}{3}$, if the value of v less than v minimum, $v < v_{min}$ then the curve will goes out of its control polygon see Figure 3.

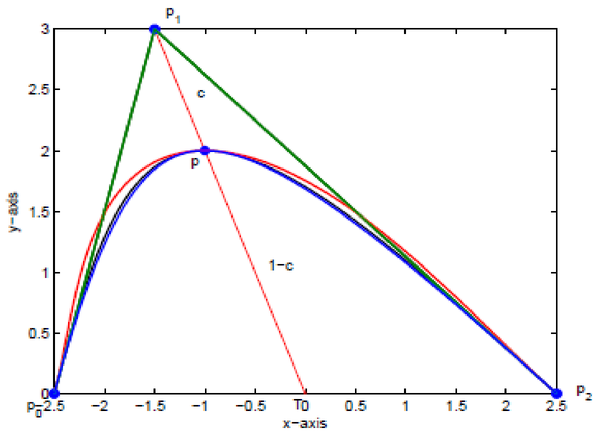

Figure 3: The value of v taken less and greater than the minimum value

Theorem 3.

Assume T be a point that divides P_0P_2 in the ratio $s : 1 - s$, and let P be a point on the line P_1T that divides it in ratio $c : 1 - c$. If the curve pass through the point P at $\theta = \frac{1}{2}$. Then the minimum value of v is $max\{v1, v2\}$, where the curve is within the control polygon $v_1 = \frac{-1+c}{c(-7+8s)}$ and $v_2 = \frac{1-c}{c(-1+8s)}$, where c, s are constants.

Proof. Let $T = (1 - s)P_0 + sP_2$, and P on the line P_1T divides in ratio $c :$ $1 - c$ then, $P = (1 - c)P_1 + cT$.

$Let\ P_0 = (t_0, s_0),\ P_1 = (t_1, s_1), P_2 = (t_2, s_2)$, then the coordinate of point $P(t, s)$ are, $P((1-c)t_1 + c((1-s)t_0 + st_2), (1-c)s_1 + c((1-s)s_0 + ss_2))$. Since the curve pass through point at $\theta = 1/2$ such that

$$(1-c)t_1 + c((1-s)t_0 + st_2) = \frac{f_0t_0 + f_1t_1v + f_2t_2}{f_0 + f_1v + f_2}$$

(6.3)

$$(1-c)s_1 + c((1-s)s_0 + ss_2) = \frac{f_0s_0 + f_1s_1v + f_2s_2}{f_0 + f_1v + f_2}$$

Where,

$$f_0 = \frac{2-\gamma}{8}, f_1 = \frac{4+\gamma+\mu}{8}, f_2 = \frac{2-\mu}{8}$$

By solving the equation (6.3) for γ and μ we get,

$$\frac{2(1-c-3cv+4csv)}{1-c+cv}$$

(6.4)

$$\mu = \frac{4(-1+c+cv)}{1-c+cv} - \frac{2(1-c+3cv+4csv)}{1-c+cv}$$

v Become minimum if $\gamma = 1, \mu = 1$. Put it in the equation (6.4) and rearrange it for v we have,

$v_1 = \frac{-1+c}{c(-7+8c)}, v_2 = \frac{1-c}{c(-1+8c)}$ Hence $v_{min} = \max\{v_1, v_2\}$

6.2 Numerical Example

Assume $v_1 > v_2$ then from the theorem 3 v_1 is minimum of v.

- If $v_2 < v < v_1$, the curve goes out of its control polygon from one side.
- If $v < v2$, the curve goes out of its control polygon from both sides.
- If $v > v1$, the curve is confined to its control polygon.

To clear this fact numerically see three Figures 4 Suppose the control points defined as $P_0 = (-4,0), P_1 = (0,7), P_2 = (5,0)$ and the value of $s = 2/3, c = 1/3$ then $v_1 = 1.2$ and $v_2 = 0.46$. That mean the minimum value of v, $v_{min} = 1.2$.

Figure 4 choosing the different value of v if we choose $v = 0.47$ the curve goes out from one side that mean the value of γ outside the domain, if v taken $v = 0.2$ the curve goes out from both side that mean the value of γ and μ outside the domain, and if we determine the value of $v = 2$ the curve is confined to its control polygon that mean the value of γ and μ inside the domain.

7. Composition of two segments of rational quadratic Bezier-Like curve.

To construct a combined curve via force particular continuity conditions by given two

segments of rational quadratic Bezier-Like curve.

Theorem 4. To achieve the G^1continuity at the point of contact of two curves $r_1(\theta)$, $r_2(\theta)$ if

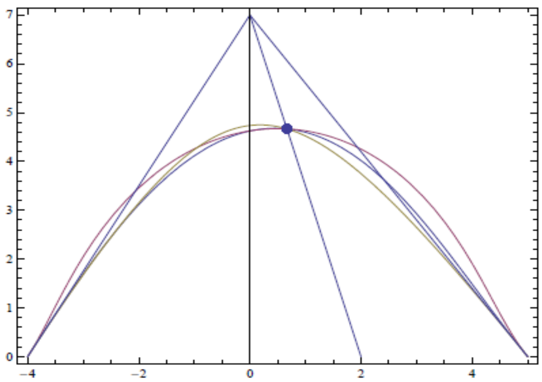

Figure 4: The different values of v

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$(P_2 - P_1) = m(Q_1 - Q_0)$ Where $m > 0$.

Proof. Suppose the two rational quadratic Bezier-Like curve be defined as:

$$r_1(\theta) = \frac{f_0 P_0 + f_1 P_1 v + f_2 P_2}{f_0 + f_1 v + f_2}, r_2(\theta) = \frac{f_0 Q_0 + f_1 Q_1 v_1 + f_2 Q_2}{f_0 + f_1 v_1 + f_2}$$

Where γ and μ are shape parameters of $r_1(\theta)$ and γ_1 and μ_1 are shape parameters of $r_2(\theta)$ with $-2 \le \gamma, \mu, \gamma_1, \mu_1 \le 1$ *and* v, v_1 are the weight for two curves.

$P_2 = Q_0$

$r'_1(1) = \tau r'_2(0)$

Substituting the value of derivatives, we have,

$$(P_2 - P_1) = \tau \frac{(2+\gamma_1)v_1}{(2+\mu)v}(Q_1 - Q_0), \tau > 0$$

$$m = \tau \frac{(2+\gamma_1)v_1}{(2+\mu)v} > 0$$

$$(P_2 - P_1) = m(Q_1 - Q_0)$$

For m = 1, the two curves provide C_1 continuity.

Theorem 5.

The curves $r_1(\theta), r_2(\theta)$ provide C^2 continuity if they satisfied C^0, C^1 continuity with the following conditions: $Q_2 = P_0 + 4v(P_1 - P_2)(1 - v - v_1)$

Where $\gamma = \mu = \gamma_1 = \mu_1 = 0$.

Proof. Let C^2 continuity, the two curves satisfied the following equation,

$$r''_1(1) = r''_2(0) \quad (7.1)$$

Setting the value of derivatives and $\gamma = \mu = \gamma_1 = \mu_1 = 0$ in equation (7.1), we get,

$$(Q_2 - Q_0) + 2v_1(1 - 2v_1)(Q_1 - Q_0) = (P_0 - P_2) - 2(P_2 - P_1)v(1 - 2v).$$

Since $(Q_1 - Q_0)v_1 = (P_2 - P_1)v$ Hence

$Q_2 = P_0 + 4v(P_1 - P_2)(1 - v - v_1)$

Theorem 6.

Let $\gamma = \mu = \gamma_1 = \mu_1 = 0$ for two curves $r_1(\theta), r_2(\theta)$ of rational quadratic Bezier-Like curve. The two curves provide G^2 continuity if they have G^0, G^1 continuity and satisfy the following conditions $H_1 = m^2 H_2, m > 0$ where H_1 is

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the distance of P_0 from the line P_1P_2, and H_2 is the distance of Q_2 From the line Q_0Q_1.

Proof. For G_2 continuity, the two curves should have same curvature at the point of contact.

$$k(0) = k(1).$$

From the definition of curvature **(Hoscheck & Lasser, 1993)**

$$\frac{r_1'(1) \times r_1''(1)}{||r_1'(1)||^3} = \frac{r_2'(0) \times r_2''(0)}{||r_2'(0)||^3}$$

The second derivative of two curves defined as the following where $\gamma = \mu = \gamma_1 = \mu_1 = 0$.

$r_1''(1) = 2((P_0 - P_2) - 2(P_2 - P_1)v + 4(P_2 - P_1)v^2)$.

$r_2''(0) = 2((Q_2 - Q_0) - 2(Q_0 - Q_1)v_1 + 4(Q_0 - Q_1)v_1^2)$.

Thus by using cross product, we get,

$$\frac{m^2|(Q_1 - Q_0)| \times |(Q_2 - Q_0)|}{||(Q_1 - Q_0)||} = \frac{|(P_2 - P_1)| \times |(P_2 - P_0)|}{||(P_2 - P_1)||}$$

Hence $$H_1 = m^2 H_2, m > 0.$$

8. Conclusion

One of the finding to emerge from this work is that the present study confirms previous finding and contributes additional evidence that suggests a new quadratic basis functions which inherits all properties of Bezier curve. This paper has shown that the two shape parameters are suitable instrument to control the shape of curve, and the presence of weights provides the designer intuitive control on the shape of the curve. The new theorems of rational quadratic Bezier-Like curve to find the minimum value of weights are presented. The two segments of rational quadratic Bezier-Like curve can joined by C^1, C^2, G^1, G^2 continuity.

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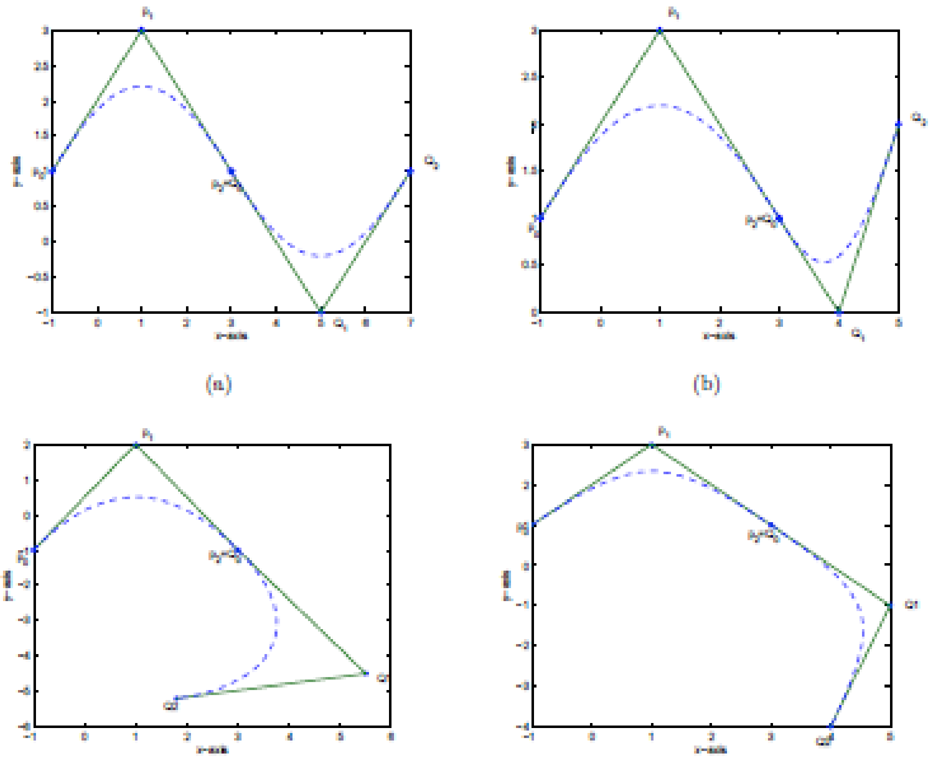

Figure 5: The composition of two segments of rational quadratic Bezier-Like curve (a) $C^1 continuity$, (b) $G^1 continuity$, (c) $C^2 continuity$, (d) $G^2 continuity$

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